MY INTELLECTUAL TRAJECTORY

Herbert E. Scarf

The time allotted for these talks is pretty short, so I won't talk about my father, Levi, who lost his small business in 1929, a few months after he married my mother, Lena Elkman. My twin brother, Frederick, and I were born in July 1930. My father never recovered from the Great Depression. We were helped out by my mother's siblings and lived in the house given to my mother's father by one of my uncles.

So let us fast-forward to puberty, a time when one fine morning I woke with the realization that I was a mathematician. I don't mean this in a formal sense; I simply grasped what mathematics was all about. I knew that a mathematical result might have several quite distinct arguments, which could be combined in a variety of ways. I knew that a theorem was different from a lemma. I read the biographies of great mathematicians, and I still have my annotated copy of *Men of Mathematics* by E. T. Bell. I taught myself the calculus of several variables and the theory of complex functions. I taught myself the first thirty-five digits of pi.

My instructors at the South Philadelphia High School for Boys—a pretty rough-house school—knew nothing about this passion of mine. In eleventh grade I learned about a Mathematics Tournament offered by Temple University for all high school students in Pennsylvania. To the shocked surprise of my teachers, and my relatives, I placed first in the tournament.

Temple University offered me a scholarship, which I accepted. As a student, I had unusual habits. I started to take graduate courses immediately. I rarely attended class. I would learn the material by myself and drop in to take the exams. The professors must have been somewhat taken aback by this behavior. But I had a wonderful piece of good luck. The single female mathematics professor at Temple, Dr. Marie Wurster, then perhaps thirty, became my friend. I was invited to knock on her door at any time that was convenient for her. We talked about mathematical topics that I was studying or planning to study; she talked about her fields of expertise. And she told me about the William Putnam Mathematical Competition, open to every undergraduate student in

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Herbert E. Scarf, Sterling Professor Emeritus of Economics, was a member of the faculty at Yale's Cowles Foundation and the Department of Economics from 1963 until his retirement. As a young boy who taught himself mathematics, Scarf astonished his teachers at South Philadelphia High School by ranking first in the 1947 Statewide Mathematical Tournament. His mathematical talent earned him a scholarship to Temple University, and he obtained his Ph.D. from Princeton University in 1954. He was especially noted for establishing the Scarf algorithm, which addressed the longstanding problem of how to find equilibrium even in the most complex economies. He was a member of the National Academy of Sciences, a fellow of the American Academy of Arts and Sciences, a member of the American Philosophical Society, a distinguished fellow of the American Economic Association, and past president of the Econometric Society. In 1973, Scarf won the Frederick W. Lanchester Prize, which recognizes the best contribution to operations research and the management sciences. A decade later, he won the John von Neumann Theory Prize for his contributions to theory in operations research.

North America. I entered the competition in my junior year at Temple and placed in the first ten.

I graduated from Temple in 1951, a year containing two extremely significant events: I met my wife and great friend, Maggie Scarf, to whom I have been married for sixty-one years. The second event was being admitted to the Graduate Department of Mathematics at Princeton in 1951.

At that time, Princeton had the very best Department of Mathematics in the entire world. My fellow students included Ralph Gomory, Lloyd Shapley, Martin Shubik, and John Nash. Nash, a recent graduate, would frequently return from MIT to Princeton. My adviser (and friend) was Salomon Bochner. My first scientific paper resulted from a remark that Bochner made in a class that I attended, and which he submitted to the *Proceedings of the National Academy of Sciences*.

The Graduate College, where I lived, was physically close to the Institute for Advanced Study, and I would frequently walk on the institute grounds. It was not unusual to see Einstein strolling with Kurt Gödel, the great logician. Einstein would smile benignly, but Gödel never did. I was totally unaware of the work being done at the institute by von Neumann and his colleagues on the modern programmable computer.

During my time at Princeton, I was disappointed by the ultra-pure mathematics that was the bread and butter of the department at that time. My hope was that the mathematical problems I worked on would have an ultimate practical application: that life would be better for someone, or some group of people, because of the intellectual issues I was struggling with.

And so it was that I left academic life and went to the RAND Corporation in Santa Monica, California, with Maggie and our newly born daughter, Martha. I was in the Mathematics department, along with George Dantzig, the inventor of the simplex method for solving linear programming problems, and Lloyd Shapley, who made the same transition as I had.

At some point, the organization suffered a budgetary crisis and I was transferred to a unit of the Economics department involved in operations research and management sciences. I started working on inventory problems: the purchase and storage of commodities whose future demands were not known with perfect certainty.

Inventory management is a serious practical field. If you are an automobile manufacturer, you don't order a door when you need it; you keep them in stock. Pharmacists keep an inventory of medicines; supermarkets keep an inventory of cheese, cans of baked beans, and boxes of linguini.

I met Kenneth Arrow, who was himself working on the management of inventories. My life was changed. He invited me to spend a year with him at Stanford jointly working on inventory theory. It was a perfect time for me. The major themes of economic theory were being formulated in mathematical terms, and I fortunately had precisely the right set of skills to make serious contributions. A lovely set of apples was hanging from the tree, and I plucked them and ate them one after another with great pleasure.

I continued to work on inventory theory and wrote a paper with Andrew Clark, then at RAND, which started the entire field of supply chain management. My most elegant result was a demonstration that the optimal policies associated with the management of inventories involved a sequence of very simple ordering policies. The simplicity of these policies in a general setting was totally unexpected. The proof did not involve very deep mathematics, but it was very odd—strange and extremely unusual. Some of my friends have said that this was the best work I had ever done.

I started to move into economic theory. Economics has a notion of a competitive equilibrium, a set of prices for *all* of the goods and services in the economy such that DEMAND = SUPPLY for all goods simultaneously. How do you find these prices if the economy is not currently at equilibrium? One idea is that prices adjust in a simple way: if the demand for a good is larger than its supply, then the price increases. If the demand is less than the supply, then the price decreases. In class, I would always refer to this as the *New York Times* adjustment mechanism. This adjustment process can be formulated mathematically as a series of differential equations. My first paper in economics was to provide a very simple example in which this process did *not* converge. The prices oscillated forever in a closed loop without approaching the equilibrium.

And so I turned my attention to the purely mathematical question of constructing an algorithm that would always find an equilibrium. The proofs of existence of a competitive equilibrium typically used Brouwer's fixed point theorem, which can be paraphrased by the statement, "A nice transformation of the unit circle, and its insides, into itself, will always leave at least one point fixed." And so, I decided to find an effective algorithm to calculate that point. And I did. To find it, google "Scarf Algorithm."

This result opened an entirely new field of economics: applied general equilibrium analysis, with the result that large general equilibrium models could be solved on the computer, more or less unrestricted by size.

I, myself, was not knowledgeable about the details of the American economy, and my original examples were small, elementary models. But I was very fortunate to have some marvelous graduate students: Timothy Kehoe, Jaime Serra, John Shoven, John Whalley, and others, who actually knew about these intricacies. They took over from me and published a number of papers containing realistic models. I quote Shoven:

> Scarf's algorithm permitted the general equilibrium model to enter the mainstream toolkit for applied economists. It removed the restrictions of being analytically tractable. Before Scarf's breakthrough, the only general equilibrium analysis of tax and trade policy was a two-sector model that could be solved analytically. Today, the models are much more disaggregated, much more sophisticated and capable of providing real guidance to economic policy makers.

Every country has its own general equilibrium model. Courses in applied general equilibrium are given throughout the world by a company called EconMod. Students in a great variety of countries are taught how to set up equilibrium models for their own economies. I am invisible to this organization.

Next I started to look at the general equilibrium model from a game theoretic point. Game theory comes in two colors: cooperative game theory and noncooperative game theory. In cooperative game theory groups of people (players) are allowed to engage in mutually beneficial activities. Alice can say to Bob, "We each own a house. I like your house more than I like mine. How do you feel?" Bob responds, "Well, I actually like yours better than mine. Let's exchange houses." And they do. I was taught about cooperative game theory by my good friend Martin Shubik, during a long walk from Columbia University to his apartment on Sutton Place.

Noncooperative game theory works with strategies. A selection of strategies, one for each player, is a Nash equilibrium if no player has an incentive to change his or her strategy, assuming that the remaining players do not change theirs. This idea has captured all of microeconomic theory. I am not delighted by it, personally.

Teaching has, of course, been an important part of my life. I prepare talks with great anxiety, even if I know their contents perfectly. I make elaborate notes, which are instantly discarded when I enter the classroom, and I begin singing from the musical score of an opera that I have in my head. I smile at all of the students and ask them rhetorical questions, which I frequently answer myself.

I really like my colleagues, though I sometimes have no idea at all about the nature of their research. In my heyday, I was on lots of university committees. I was director of the Division of Social Sciences. Charles Taylor, the provost of those years, once asked me to join the provost's office. He had not yet realized an important aspect of my personality: *I cannot make things actually happen in the real world!*

I adore my family: I have three daughters, with three respective spouses, seven grandchildren, and one great-grandchild. I am told by many, many people that my lovely wife is a wonderful writer. I believe them completely, and I have always enjoyed being her first editor. Maggie and I have seen a great many operas and we love ballet. We have, for many years, had the same wonderful seats at the Met for the American Ballet. We also go to the movies in Manhattan, and we have some favorite greasy spoons near our apartment on West 66th Street near the Met.

Back to economics. The general equilibrium model has two major failings, but despite these failings, it cannot be ignored.

The model doesn't treat time very well. If you make a consumption or investment decision today, it would be very useful to know your income tomorrow, and more generally the way in which the economy is going to develop over time. This leads to macroeconomics.

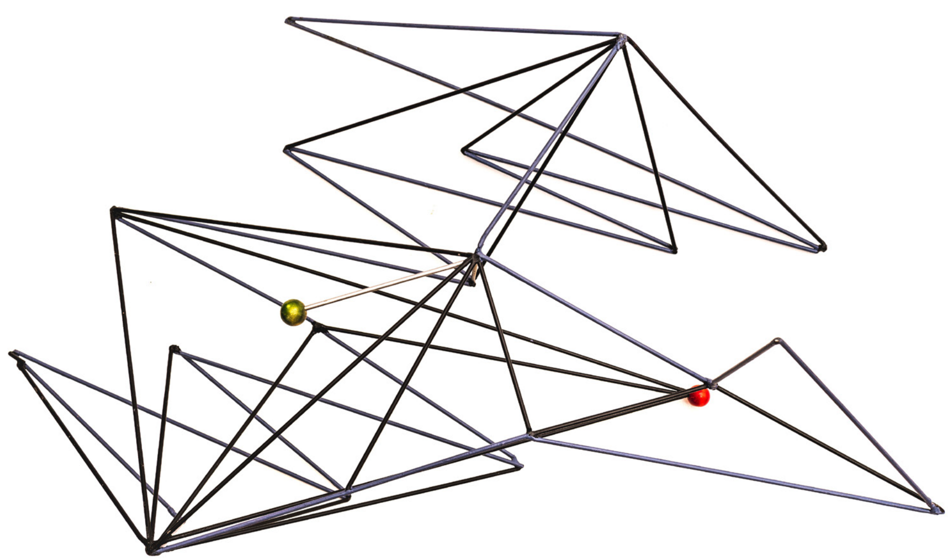
What is the second failing? It may not be clear to the general public that economic theory has no way of dealing with economies of scale in production—I mean *no way at all.* Economic theory makes the assumption of constant returns to scale. Taken literally, this means that if I want an automobile, I would purchase steel, glass, rubber, electrical wiring, and tools and hire skilled labor and produce the car in my own backyard. The assembly line is irrelevant to classical economic theory. I think about this issue a lot. It deals with *indivisibilities in production:* large items like an assembly line, a bridge, the railroad track from New York to Boston, the underwater Channel Tunnel from Britain to France.

And so I busied myself in this research. On the way, I wrote complex and elegant papers, some of which started new areas of research. Indeed, you might consider me to be an intellectual polygamist, because I am considered the father of several important fields in economics, computer science, and operations research. One of these fields led to the beautiful figure being passed around the room. The figure was made by an eminent sculptress, Ann Lehman, who is in the audience.

What is this figure? It is a collection of lines in space. The ends of the lines are points in space and, as such, have coordinates (x, y, z). In this case the points have integral coordinates, such as (7, 1, 3).

There actually are triangles in the figure: any three edges that look like a triangle are actually a triangle in the figure. We didn't put them in the sculpture, because they would obscure the lines. So the figure is really a two-dimensional surface made up of lots of triangles. It's a very nice figure: a circle and its interior. The red ball is the center of the circle; the other points are on the boundary.

The boundary points are connected by the blue line. There are many other lines which are *not* on the boundary. The green point is not in the figure: it's a reference point. This figure is the essential part of an algorithm for solving a set of mathematical problems of enormous practical significance.



Ann P. Lehman, *Scarf Complex,* 2013. Stainless steel